A SYSTEMS EFFECTIVENESS MODEL FOR AN ALTERNATIVE STRUCTURED COMPLEX SYSTEM

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THESIS

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ABSTRACT

A complex system envisioned can accomplish a given mission with the aid of any one of several alternative structures. Each of these alternative structures (subsystems), although capable of mission accomplishment, exhibits different levels of performance due to its component inventory. The structures are ordered in preference accordingly. Operation on less-preferred structures is a function of component/structure failure histories in prior structures. Failures are classified by three levels of severity. A system effectiveness model is developed to provide a measure of effectiveness for the overall complex system which encompasses all possible alternative structures.

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TABLE OF SYMBOLS AND ABBREVIATIONS

Symbol	Meaning
S_j	Alternative structure j from the admissible set of alternative structures $\{S_j, j = 1,2,\cdots, J\}$ for mission m
C_{jg}	Critical failure event in S_j
M_{jk}	Major failure event in S_j
$m_{j\ell}$	Minor failure event in S_j
$\mathcal{F}_j$	$\{C_{jg}, g = 1,2,\cdots, G_j\}$, the set of all critical failures in S_j
$\mathcal{M}_j$	$\{M_{jk}, k = 1,2,\cdots, K_j\}$, the set of all major failures in S_j
$\mathcal{K}_j$	$\{m_{j\ell}, \ell = 1,2,\cdots, L_j\}$, the set of all minor failures in S_j
T	Mission m duration time
$\lambda_{jg}, \lambda_{jk}, \lambda_{j\ell}$	Failure rates associated with C_{jg}, M_{jk}, and $m_{j\ell}$, respectively
$t_{jg}, t_{jk}, t_{j\ell}$	Operation time during mission associated with C_{jg}, M_{jk}, and $m_{j\ell}$, respectively
$D_{jk,j+1}$	Degradation factor on effectiveness of structure S_{j+1} from the major failure M_{jk} in S_j
$d_{j\ell;j}$	Degradation factor on effectiveness of structure S_j from the minor failure $m_{j\ell}$ in S_j

$RC_j(t)$	Probability of no critical failures in S_j during the time interval $(0,t)$
$RM_j(t)$	Probability of no major failures in S_j during the time interval $(0,t)$
E_j^*	Alternative structure S_j effectiveness (Probability of mission success on S_j) given no failures
E_j'	Average effectiveness of S_j accounting for major failures in structures $S_1, S_2, \ldots, S_{j-1}$
E_j	Average of E_j' accounting for minor failures in S_j
$X_{j\ell}$	Time to event $m_{j\ell}$, minor failure in component ℓ in S_j
$T_{j'k}$	Time to event $M_{j'k}$, major failure in component k in $S_{j'}$
$Y_{j'}$	Min $\left\{T_{j'1}, T_{j'2}, \ldots, T_{j'K_{j'}}\right\}$, the time to first major failure in $S_{j'}$
Σ_j	$\sum_{j'=1}^{j} Y_{j'}$, the minimum time for a sequence of major failures in structures $S_1, S_2, \ldots, S_j$
f_{Σ_j}	P.D.F. for minimum time for a sequence of major failures in structures $S_1, S_2, \ldots, S_j$

λ_j	$\sum_{k=1}^{K_j} \lambda_{jk}$, summation of major failure rates for components in S_j
$\bar{\lambda}_j$	$\frac{1}{j} \sum_{i=1}^{j} \lambda_i$, approximated mean time to major failure in a sequence of major failures in $S_1, S_2, \cdots, S_j$
$RC_{j+1}(T-t)$	Probability structure S_{j+1} experiences no critical failures in remaining mission time (T-t)
$RM_{j+1}(T-t)$	Probability structure S_{j+1} experiences no major failures in remaining mission time (T-t)
P_j	Probability structures $S_1, S_2, \cdots, S_j$ experience major failures and no critical failures, and S_{j+1} experiences no major or critical failures in (0,T)
E_m	System effectiveness on mission m accounting for all reliable alternative structures.

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CHAPTER I

INTRODUCTION

In light of the present day advancing technology, systems have become vastly complex. Coupled with this rise in complexity has been a gradual increase in component and system size, weight and/or cost. The rise in complexity in just avionics systems over the 12-year period between 1953 and 1965 has been by a factor of 74, not to mention the expected complexity factor of 160 times the 1953 avionic system for the year 1968. [1] None-the-less these new complex systems cannot always incorporate redundant components or subsystems due to size, weight, and/or cost constraints; this is particularly true for an aircraft. However, there may exist a less-sophisticated component or group of components that can perform the same (or similar) function as a complex component, but somehow in a less effective manner, e.g., solution time longer, greater uncertainty in solution, or lower probability of success. This less effective group of components, "alternative structure", can also accomplish the mission and thereby improves the overall system effectiveness by a new form of redundancy, since the usual form of redundancy implies equal effectiveness.

The foregoing concerning complex systems and associated constraints applies most assuredly to military weapons systems being contemplated for aircraft, ship and submarine platforms of the future.

I. PURPOSE OF THE SYSTEMS EFFECTIVENESS MODEL

In spite of the enormous complexities and the size, weight and/or cost constraints, a weapons system must still achieve a high mission reliability, usually in the high nineties, where mission reliability is defined to be "the probability of adequate system performance for the duration of the mission in the usual mission environment." In this paper adequate system performance will imply that at least one alternative structure, i.e., a subsystem capable of mission accomplishment, remains operable. An operable structure by definition experiences only repairable type (minor) failures during a mission.

The problem is to develop a model to measure the overall system effectiveness which incorporates the effects of availability, conditioned effectiveness and the component/structure failure degradation over all admissible alternative structures. The measure of system effectiveness derived, hopefully, will approximate the "systems performance effectiveness" defined as "a measure of the extent to which a system can be expected to complete its assigned mission within the time frame under the stated environmental conditions." [2]

II. SIGNIFICANCE OF THE MATHEMATICAL MODEL

The United States Navy currently has under development several new complex weapons systems to cope with the growing submarine threat. The P3C patrol aircraft, being developed by Lockheed at Burbank, California, and under evaluation by the ASW Projects Office, Washington, D.C., is one such weapons system. The revolutionary weapons system incorporates a variety of sophisticated detection sensors in a complex computerized system design. For each of the several missions of the P3C aircraft, there exists a set of alternative structures of components feasible to the accomplishment of that mission. The ultimate purpose of this effectiveness model is to obtain a quantitative measure to be used in design alteration comparisons in the P3C aircraft.

The alternative structured approach for the P3C aircraft seems practicable for the following reasons: (1) by the nature of the aircraft/submarine detection and tracking problem, the structure most effective for a mission may change during a mission engagement as a result of environmental or target tactics, and (2) by reason of limited weight and space requirements aboard an aircraft, a failure may occur to cause an operating structure to be inoperative (non-repairable) for the remainder of the mission. Either of these reasons necessitates operation on another alternative structure, usually of different performance and/or reliability.

The mathematical model developed by Zagor, et al., [3] for computing the reliability of a hypothetical multi-model fire control system treated a similar system redundancy variation. In the Zagor model the system effectiveness of a three mode hypothetical fire control system was formulated accounting for mode effectiveness and mode likelihood. The system effectiveness model to be described in Chapter II and derived mathematically in Chapter III takes into consideration three levels of failure severity: failures unacceptable to mission accomplishment, failures permissible but non-repairable, and failures permissible and repairable.

CHAPTER II

DESCRIPTION AND FORMULATION OF MODEL

A particular mission of a complex system can be accomplished by any one of several alternative structures (subsystems) of the system components. The alternative structures are composed of components in logical series and are ordered in preference by their attainable degree of mission performance. Component failures in this complex system are grouped into three classes of severity, namely critical, major and minor failures. Component failures that are not allowable in the accomplishment of the mission, and as such cause mission abort when they occur, are defined as critical failures. A major failure is one whose occurrence precludes any further operation with the associated structure during mission, i.e., forces a shift to a lower priority structure, and may result in degrading the effectiveness of an alternative structure. Component failures repairable during a mission by on-board repair parts are defined as minor failures. Minor failures degrade structure/system effectiveness (perhaps proportional to structure down-time or function lost) but do not require a shift to another alternative structure. This model establishes a functional relationship to quantify the overall system effectiveness taking into consideration the various performance levels and reliabilities of all mission-orientated alternative structures.

The primary or priority one alternative structure is that structure capable of obtaining the greatest degree of mission performance due to its sophisticated component design. The priority one structure, S_1, may in a particular system include all components of the system. However, this is usually not the case since most complex systems incorporate redundant components (components that will perform the same function) having a lower performance capability. The performance of the priority one structure in a failure-free environment is primarily a function of three variables, namely: (1) the state of the art in component/system engineering design, (2) the environmental operating factors encountered during a mission, and (3) the component operator's state of training and performance levels. However, there exists some average level of effectiveness, E_1^*, realized when operating the system utilizing the primary structure S_1 in a failure-free environment. But when failures occur, structure/system effectiveness becomes degraded.

For a particular mission a complex system has the set of admissible alternative structures $\{S_j, j = 1,2,\ldots, J\}$ ordered in preference. Each alternative structure S_j has associated with it the sets of components susceptable to critical, major and minor failures, i.e., $\mathcal{F}_j$, $\mathcal{M}_j$ and $\mathcal{K}_j$ respectively, where

$$\mathcal{F}_j = \left\{C_{j1}, C_{j2}, \cdots, C_{jg}, \cdots, C_{jG_j}\right\}$$

$$\mathcal{M}_j = \left\{M_{j1}, M_{j2}, \cdots, M_{jk}, \cdots, M_{jK_j}\right\}$$

$$\mathcal{K}_j = \{m_{j1}, m_{j2}, \ldots, m_{j\ell}, \ldots, m_{jL_j}\}$$

and $C_{j\cdot}$, $M_{j\cdot}$, $m_{j\cdot}$ represent the events of critical, major and minor failures in structure S_j, respectively. Similarly, λ_{jg}, λ_{jk}, $\lambda_{j\ell}$ and t_{jg}, t_{jk}, $t_{j\ell}$ correspond to the component failure rates and operating durations for the components in $\mathcal{F}_j$, $\mathcal{M}_j$ and $\mathcal{K}_j$, respectively. The effectiveness of a system experiencing a critical (abort type) failure will be by definition equal zero. The major failures are those failures which are non-repairable during the mission and which force system operation into an alternative structure of less desirable performance. This less effective structure is also susceptible to degradation from the major failure(s) in prior structure(s). Let the degradation factor, $D_{jk,j+1}$, be defined as the degradation factor on structure S_{j+1} resulting from event M_{jk} in structure S_j. Minor failures also connote a degrading effect, but the degradation factor is applied to present structure effectiveness. Now $d_{j\ell,j}$ denotes the degradation factor on structure S_j due to the event $m_{j\ell}$ in S_j. Several assumptions are now made about the component failure distributions.

Independent exponential failure time distributions are assumed on all components and on all subassemblies whose failure is a critical, major, or minor event. The failure rate, λ, of a component is the same regardless of the alternative structure in which it is operating. The failure rates of components in non-operating states are equal to zero. If a particular component in the system is susceptible to more than one class of failure, then that component will be subdivided into pseudo components affected by only one class of failure for ease of formulation of the model. The effect of failures on mission performance will now be considered.

From the preceding discussion of failure types and considering a particular system design, one can enumerate the success paths (system failure histories) such that the mission will still be accomplished, but degraded in varying amounts by the effects of associated failures. To obtain a quantified measure of effectiveness that incorporates the flexibility in performance and reliability in an alternative structured system, the system effectiveness model is formulated below encompassing all possible success paths in the accomplishment of the mission.

CHAPTER III

SYSTEM EFFECTIVENESS MODEL DERIVATION

Mission accomplishment is associated with the greatest system effectiveness when no critical or major failures occur during the mission duration (0,T). In this particular "success path", i.e., no failure in the primary structure S_1, the overall system effectiveness is the product of E_1 (the average effectiveness of structure S_1 accounting for minor failure degradation effects) times the probability of no critical or major failures. Let $RC_j(t)$ and $RM_j(t)$ be respectively the probability of no critical and of no major failures in structure S_j during time (o,t). Then, if only one structure existed for a particular system, the system effectiveness would be

$$E = E_1 \times RC_1(T) \times RM_1(T).$$

However, in the complex system envisioned here, there are several alternative structures feasible to the accomplishment of mission, and the alternative structures are utilized when preferred structures fail. As such, system effectiveness must account for this incremental effectiveness obtained via alternative structures when these same structures are probable.

The formulation of such a model accounting for the effectiveness of all alternative structures weighted by each structure's probability of being utilized follows:

System Effectiveness[†] on Mission m (E_m) is modeled as

$$E_m = E_1 \times RC_1(T) \times RM_1(T) + \sum_{j=2}^{J} E_j \, P_{j-1} \tag{1}$$

where

$RC_1(T)$ = Probability of no critical failures in S_1 during mission

$$= \exp\left(- \sum_{g=1}^{G_1} \lambda_{1g} \, t_{1g}\right)$$

$RM_1(T)$ = Probability of no major failures in S_1 during mission

$$= \exp\left(- \sum_{k=1}^{K_1} \lambda_{1k} \, t_{1k}\right)$$

E_j = Average structure S_j effectiveness accounting for degradation from major failures in prior structures and degradation from minor failures in S_j

and

P_j = Probability structure S_1, S_2, ..., S_j each experience a major and no critical failure, and S_{j+1} experiences no major or critical failures in (O,T)[‡]

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† Accounts for all (J) admissible alternative structure effectiveness given the structure is reliable.

‡ During mission hereafter denoted by (O,T).

Now E_j is obtained in the following manner. First of all, it is assumed that the effectiveness of performance level of any particular alternative structure given no failures, defined as E_j^*, is known or may be estimated by operational exercises. The E_j^* represents the nominal structure S_j effectiveness for the current state of the art, the environmental operating factors encountered, and the operator performance level as discussed previously. But E_j^* must account for (i.e., be degraded by, when appropriate) the effect of major failures in prior structures $S_1, S_2, \ldots, S_{j-1}$ on structure S_j. Letting $D_{ik,j}$ be the degradation factor† on S_j from the first major failure M_{ik} in structure S_i, for $i < j$, it follows that

$$E_j' = E_j^* \sum_{i=1}^{j-1} \sum_{k=1}^{K_i} D_{ik,j} \left(\begin{array}{c}\text{Probability first major failure} \\ \text{in } S_i \text{ is } M_{ik}\end{array}\right)$$

$$= E_j^* \sum_{i=1}^{j-1} \sum_{k=1}^{K_i} D_{ik,j} \left(\frac{\lambda_{ik}}{\sum_{k'=1}^{K_i} \lambda_{ik'}} \right) \qquad \text{for } j = 2,3,\ldots,J$$

where E_1' is identically E_1^*, and

E_j' = Average effectiveness of S_j accounting for major failures in structures $S_1, S_2, \ldots, S_{j-1}$.

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† When a major failure has no degradation effect on a structure, then the degradation factor is zero.

where the term in parentheses above is derived in Appendix A.

A particular alternative structure S_j effectiveness should also take into consideration the several success paths under various minor failure histories within that structure. Therefore a weighted average of the E_j', defined above, where the weights are the products of the various probabilities of minor failure(s) and the associated minor failure(s) degradation seems plausible for an average structure effectiveness E_j capability. Formulating the above we obtain, assuming independent exponential distributions, the following:

$$E_j = E_j' \left(\text{Probability no minor failures in } S_j \text{ in } (0,T)\right)$$
$$+ E_j' \sum_{\ell'=1}^{L_j} \left(d_{j\ell',j}\right) \begin{pmatrix}\text{Prob. } m_{j\ell'} \text{ occurs} \\ \text{in } S_j \text{ in } (0,T)\end{pmatrix} \begin{pmatrix}\text{Prob. no other} \\ \text{minor failures in} \\ S_j \text{ in } (0,T)\end{pmatrix}$$
$$+ E_j' \sum_{\ell'=1}^{L_j} \sum_{\substack{\ell''=1 \\ \ell''>\ell'}}^{L_j} \left(d_{j\ell',j}\right)\left(d_{j\ell'',j}\right) \begin{pmatrix}\text{Prob. } m_{j\ell'} \\ \text{and } m_{j\ell''} \\ \text{occur in} \\ S_j \text{ in } (0,T)\end{pmatrix} \begin{pmatrix}\text{Prob. no} \\ \text{other minor} \\ \text{failures in} \\ S_j \text{ in } (0,T)\end{pmatrix}$$
$$+ \epsilon \quad \text{(small order terms)}$$

Inserting the probabilistic results derived in Appendix B, it follows that†

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† A more conservative approximation would be obtained by replacing $t_{j\ell}$ in the first term by T; also simplifying calculations.

$$E_j = E_j' \left[e^{-\sum_{\ell=1}^{L_j} \lambda_{j\ell} t_{j\ell}} \right]$$

$$+ E_j' \sum_{\ell'=1}^{L_j} \left(d_{j\ell',j}\right) \left(1 - e^{-\sum_{\ell'=1}^{L_j} \lambda_{j\ell'} T}\right) \left(e^{-\sum_{\substack{\ell=1 \\ \ell \neq \ell'}}^{L_j} \lambda_{j\ell} T}\right)$$

$$+ E_j \sum_{\ell'=1}^{L_j} \sum_{\substack{\ell''=1 \\ \ell''>\ell'}}^{L_j} \left(d_{j\ell',j}\right)\left(d_{j\ell'',j}\right) \left(1-e^{-\sum_{\ell'=1}^{L_j} \lambda_{j\ell'} T}\right) \left(1-e^{-\sum_{\ell''=1}^{L_j} \lambda_{j\ell''} T}\right) \left(e^{-\sum_{\substack{\ell=1 \\ \ell \neq \ell', \ell''}}^{L_j} \lambda_{j\ell} T}\right)$$

$+ \epsilon$ (small order terms)

Simplification to the calculation of the E_j terms, when the $\sum_{\ell=1}^{L_j} \lambda_{j\ell}$ exponent terms are small, would be to use approximation

$$\left(1-e^{-\sum_{\ell=1}^{L_j} \lambda_{j\ell} T}\right) \approx \sum_{\ell=1}^{L_j} \lambda_{j\ell} T + \frac{\left(\sum_{\ell=1}^{L_j} \lambda_{j\ell} T\right)^2}{2} + \cdots$$

discarding the higher order terms when input data shows them to be insignificant.

P_j, the probability that structures $S_1, S_2, \cdots, S_j$ each experience a major and no critical failure, and structure S_{j+1} experiences no major or critical failures in $(0,T)$ is obtained as follows. Let $Y_{j'}$ be the minimum $\left\{T_{j'1}, T_{j'2}, \cdots, T_{j'K_{j'}}\right\}$ where $T_{j'k}$ is the time to event $M_{j'k}$ (major failure event in $S_{j'}$), then $Y_{j'}$ represents the time to first major failure in $S_{j'}$. Now define

$$\Sigma_j \equiv \sum_{j'=1}^{j} Y_{j'}$$

as the "time for a sequence of major failures in structures $S_1, S_2, \ldots, S_j$", with the assumption of zero switching time between alternative structures. Let $f_{\Sigma_j}(t)$ be the probability density function of the time for a sequence of major failures in structures $S_1, S_2, \ldots, S_j$. Thus

$$P_j = \text{Prob.}\left\{\begin{array}{l} S_1, S_2, \ldots, S_j \text{ fail major,} \\ S_1, S_2, \ldots, S_j \text{ not fail critical,} \\ S_{j+1} \text{ not fail major for remainder of mission,} \\ S_{j+1} \text{ not fail critical for remainder of mission} \end{array}\right\}$$

and

$$P_j = \int_0^T f_{\Sigma_j}(t) \left[\prod_{j'=1}^{j} RC_{j'}(t)\right] \left[RM_{j+1}(T-t)\right] \left[RC_{j+1}(T-t)\right] dt \tag{5}$$

where

$$f_{\Sigma_j}(t) \doteq \frac{(\bar{\lambda}_j)^j t^{j-1}}{\Gamma(j)} e^{-\bar{\lambda}_j t} \qquad j = 1, 2, \ldots, J-1 .$$

is a gamma P.D.F. [4], where equal structure $\bar{\lambda}_j$ (mean time to first major failure is assumed) is

$$\bar{\lambda}_1 \equiv \lambda_1 \,, \qquad \text{where} \qquad \lambda_j = \sum_{k=1}^{K_j} \lambda_{jk}$$

$$\bar{\lambda}_2 = (\lambda_1 + \lambda_2)/2$$

$$\vdots$$

$$\bar{\lambda}_j = (\lambda_1 + \lambda_2 + \cdots + \lambda_j)/j$$

Assuming that $\prod_{j'=1}^{j} RC_{j'}(t) \cong \prod_{j'=1}^{j} RC_{j'}(T)$, then Eq. (5) becomes

$$P_j \cong \prod_{j'=1}^{j} RC_{j'}(T) \int_o^T \left[\frac{(\bar{\lambda}_j)^j \, t^{j-1}}{\Gamma(j)} \, e^{-(\bar{\lambda}_j)t} \right] \left[e^{-\sum_{k=1}^{K_j} \lambda_{jk}(T-t)} \right] \left[e^{-\sum_{g=1}^{G_j} \lambda_{jg}(T-t)} \right] dt \tag{6}$$

Examining Eq. (6) for a few small j values, we obtain

$$P_1 = e^{-\sum_{g=1}^{G_1} \lambda_{1g} t_{1g}} \int_o^T \left[\lambda_1 e^{-\lambda_1 t} \right] \left[e^{-\sum_{k=1}^{K_2} \lambda_{2k}(T-t)} \right] \left[e^{-\sum_{g=1}^{G_2} \lambda_{2g}(T-t)} \right] dt$$

$$= e^{-\left[\sum_{g=1}^{G_1} \lambda_{1g} t_{1g} + \sum_{g=1}^{G_2} \lambda_{2g} T + \sum_{k=1}^{K_2} \lambda_{2k} T \right]} \int_o^T \lambda_1 e^{-\left[\lambda_1 - \sum_{k=1}^{K_2} \lambda_{2k} - \sum_{g=1}^{G_2} \lambda_{2g} \right] t} dt$$

$$= e^{-D_1} \int_o^T \lambda_1 e^{-F_1 t}\, dt$$

where

$$D_1 = \left[\sum_{g=1}^{G_1} \lambda_{1g} t_{1g} + \sum_{g=1}^{G_2} \lambda_{2g} T + \sum_{k=1}^{K_2} \lambda_{2k} T \right]$$

$$F_1 = \left[\lambda_1 - \sum_{k=1}^{K_2} \lambda_{2k} - \sum_{g=1}^{G_2} \lambda_{2g} \right]$$

Then it follows that

$$P_2 = e^{-D_2'} \int_o^T \left[\frac{(\bar{\lambda}_2)^2 t}{\Gamma(2)} e^{-\bar{\lambda}_2 t} \right] \left[e^{-\sum_{k=1}^{K_3} \lambda_{3k}(T-t)} \right] \left[e^{\sum_{g=1}^{G_3} \lambda_{3g}(T-t)} \right] dt$$

$$= e^{-D_2} \int_o^T (\bar{\lambda}_2)^2\, t \cdot e^{-\left(\bar{\lambda}_2 - \sum_{k=1}^{K_3} \lambda_{3k} - \sum_{g=1}^{G_3} \lambda_{3g} \right) t}\, dt$$

$$= e^{-D_2} \int_o^T (\bar{\lambda}_2)^2\, t\, e^{-F_2 t}\, dt$$

where

$$D_2' = \sum_{g=1}^{G_1} \lambda_{1g} t_{1g} + \sum_{g=1}^{G_2} \lambda_{2g} t_{2g} = \sum_{i=1}^{2} \sum_{g=1}^{G_i} \lambda_{ig} t_{ig}$$

$$D_2 = D_2' + \sum_{g=1}^{G_3} \lambda_{3g} T + \sum_{k=1}^{K_3} \lambda_{3k} T$$

$$F_2 = \bar{\lambda}_2 - \sum_{k=1}^{K_3} \lambda_{3k} - \sum_{g=1}^{G_3} \lambda_{3g}$$

Similarly we obtain P_3

$$P_3 = e^{-D_3} \int_0^T \frac{(\bar{\lambda}_3)^3 \; t^2}{\Gamma(3)} e^{-F_3 t} \, dt$$

Each of the integral expressions in P_1, P_2 and P_3 may be transformed into incomplete gamma functions (P_1 degenerates into an exponential function) as follows

$$P_1 = e^{-D_1} \int_0^T \lambda_1 e^{-F_1 t} \, dt = e^{-D_1} \left(\frac{\lambda_1}{F_1} \right) \int_0^T F_1 e^{-F_1 t} \, dt$$

$$P_2 = e^{-D_2} \int_0^T (\bar{\lambda}_2)^2 \; t e^{-F_2 t} \, dt = e^{-D_2} \frac{(\bar{\lambda}_2)^2}{(F_2)^2} \int_0^T (F_2)^2 t e^{-F_2 t} \, dt$$

$$P_3 = e^{-D_3} \left(\frac{\bar{\lambda}_3}{F_3} \right)^3 \int_o^T \frac{(F_3)^3 t^2}{\Gamma(3)} e^{-F_3 t} \, dt$$

$$= e^{-D_3} \left(\frac{\bar{\lambda}_3}{F_3} \right)^3 \int_o^T \frac{s^2}{\Gamma(3)} e^{-s} \, ds$$

The latter is obtained by the substitution, $s = F_3 t$.

The Incomplete Gamma Function is evaluated from Incomplete Gamma tables or readily from Poisson tables, after using the following well known identity (See Appendix C).[5]

$$\int_o^T \frac{(\lambda t)^{j-1}}{\Gamma(j)} e^{-\lambda t} \, dt \equiv \sum_{i=j}^{\infty} \frac{(\lambda T)^i}{(i)!} e^{-\lambda T}$$

For example

$$P_3 = e^{-D_3} \left[\frac{\lambda_3}{F_3} \right]^3 \sum_{i=3}^{\infty} \frac{(F_3 T)^i}{i!} e^{-F_3 T}$$

Now that all terms of equation (1) have been formulated, each may be evaluated with proper input and inserted in Eq.(1) to give a quantitative measure of overall system effectiveness.

CHAPTER IV

SUMMARY AND CONCLUSIONS

I. SUMMARY

"The effective performance of Navy systems in fleet use is essential to successful Navy operations." [1] The alternative structure concept of a complex weapons system seems to have merit in the improvement of "system performance effectiveness." The P3C aircraft contains a complex system that is recognized to have such a structure due to the nature of its mission.

In the preceding chapters a complex mission oriented system composed of a set of ordered alternative structures (any one of which are capable of mission accomplishment) was described. The preference ordering of structures was assumed to be by performance. Failures of components of a structure were classified into one of three levels of severity: critical, major or minor failure. The effects of failures on system effectiveness were accounted for by degradation factors, such that a critical failure caused total degradation or mission abort, a major failure completely degraded the current structure, and a minor failure (repairable) partially degraded the current structure.

A mathematical model for the above system design was then developed to obtain a quantitative measure of the overall system effectiveness reflecting the capability or multiple success paths admissible in the accomplishment of a particular mission.

II. CONCLUSIONS

Several assumptions and approximations (presumably conservative) were made in obtaining a closed form solution. In any application of the model the assumptions and limitations must be considered. First of all, the model applies to a specific kind of system design. The measure of effectiveness obtained from the model can only be as good as the input data; consequently, failure rates, performance and degradation factors, etc., should be revised to reflect current knowledge. If independent exponential failures cannot be assumed, then the model must be modified or used with discretion. When the assumption can be accepted, this model affords a straight-forward solution with the proper input paramters.

III. ADDITIONAL RESEARCH

Further research effort is needed to test the model (for example by simulation techniques) for its sensitivity, with current P3C design parameters. While this model treats a major failure as a trigger to cause a shift in alternative structure, the model could be altered to address the problem of a structure shift due to a target tactical maneuver. This preliminary model when validated could by simulations provide insight into the effects of changing onboard repair parts and altering a component structure failure severity by incorporation of redundant components. In this way, design revision results in terms of "system performance effectiveness" could be quantified for decision purposes.

BIBLIOGRAPHY

1. Navy Systems Performance Effectiveness Manual, NAVMAT P3941-A, compiled by Headquarters Naval Material Command, Washington, D. C., July, 1968.

2. Systems Effectiveness, compiled by Systems Effectiveness Branch, Office of Naval Material, January, 1965.

3. Zagor, Herbert I., et al., "Reliability of Multi-Moded Systems," Electronic Industries, Vol. 17, pt. I, April, 1958.

4. Cox, Davis R., Renewal Theory, New York: John Wiley & Sons, Inc., 1962.

5. Parzen, Emanuel, Modern Probability Theory and Its Applications, New York: John Wiley & Sons, Inc., 1960.

APPENDIX A

1. Derivation of Probability $\begin{pmatrix}\text{First major failure} \\ \text{in } S_i \text{ is } M_{ik}\end{pmatrix}$

Let T_{ik} = the random variable time to major failure M_{ik}.

$T_{ik} \sim \exp(\lambda_{ik})$

$V = \underset{\substack{1 \le k' \le K_i \\ (k' \ne k)}}{\text{Min}} \quad T_{ik'}$

then

$$1 - F_V(t) = \text{prob.}(V>t) = \frac{P(T_{i1}>t)P(T_{i2}>t)\cdots P(T_{iK_i}>t)}{P(T_{ik}>t)}$$

$$= \exp\left(- \sum_{\substack{k'=1 \\ (k' \ne k)}}^{K_i} \lambda_{ik'} t \right)$$

and

$$f_V(t) = \lambda_{k'} e^{-\lambda_{k'} t} \quad \text{where} \quad \lambda_{k'} \equiv \sum_{\substack{k'=1 \\ k' \ne k}}^{K_i} \lambda_{ik'}$$

Now

$$\text{Prob.}\begin{pmatrix}\text{First major failure} \\ \text{in } S_i \text{ is } M_{ik}\end{pmatrix} = \text{Prob.}\left(T_{ik} < V\right)$$

$$= \int_0^\infty \lambda_{ik} e^{-\lambda_{ik} T_{ik}} \int_{T_{ik}}^\infty \lambda_{k'} e^{-\lambda_{k'} V} dV dT_{ik}$$

$$= \int_0^\infty \lambda_{ik} e^{-(\lambda_{ik} + \lambda_{k'}) T_{ik}} dT_{ik}$$

$$= \frac{\lambda_{ik}}{\lambda_{ik} + \lambda_{k'}}$$

$$= \frac{\lambda_{ik}}{\sum_{k'=1}^{K_i} \lambda_{ik'}}$$

APPENDIX B

1. Derivation of Probability $\begin{pmatrix} \text{No minor failures} \\ \text{in } S_j \text{ in } (0,T) \end{pmatrix}$

Let $X_{j\ell}$ = the random variable time minor failure

$\lambda_{j\ell}, t_{j\ell}$ = failure rate and operation duration of the component ℓ in structure S_j

$$\text{Prob.}\begin{pmatrix} \text{No minor failures} \\ \text{in } S_j \text{ in } (0,T) \end{pmatrix}$$

$$= \text{Prob.}\left(X_{j\ell} > t_{j\ell};\quad \text{for all } 1 \le \ell \le L_j\right)$$

$$= \exp\left(-\sum_{\ell=1}^{L_j} \lambda_{j\ell} t_{j\ell}\right)$$

2. Derivation of Prob. $\begin{pmatrix} m_{j\ell'} \text{ occurs} \\ \text{in } S_j \\ \text{in } (0,T) \end{pmatrix}$ Prob. $\begin{pmatrix} \text{No other minor} \\ \text{failures in} \\ S_j \text{ in } (0,T) \end{pmatrix}$

$$= \text{Prob.}\left(X_{j\ell'} < T\right) \text{Prob.}\left(\min_{\ell \ne \ell'} X_{j\ell} > T\right)$$

$$= \left(1 - e^{-\lambda_{j\ell'}T}\right) e^{\left(-\sum_{\substack{\ell=1 \\ \ell \ne \ell'}}^{L_j} \lambda_{j\ell} T\right)}$$

3. Derivation of Prob. $\begin{pmatrix} m_{j\ell'}, m_{j\ell''} \\ \text{occur in} \\ S_j \text{ in } (0,T) \end{pmatrix}$ Prob. $\begin{pmatrix} \text{No other minor} \\ \text{failures in} \\ S_j \text{ in } (0,T) \end{pmatrix}$

follows similarly and is given by

$$= \left(1 - e^{-\lambda_{j\ell'}T}\right)\left(1 - e^{-\lambda_{j\ell''}T}\right) e^{\left(-\sum_{\substack{\ell=1 \\ \ell\neq\ell',\ell''}}^{L_j} \lambda_{j\ell}T\right)}$$

APPENDIX C

1. Show $\int_0^T \frac{(\lambda t)^{j-1}}{\Gamma(j)} e^{-\lambda t}\, dt \equiv \sum_{i=j}^{\infty} \frac{(\lambda T)^i}{i!} e^{-\lambda T}$

where λ is the mean rate of occurrence of failures.

Let $F_j(T)$ = Probability that the time to occurrence of the j^{th} major failure event will be less than T(represented by the term on left of the identity above).

then, $1 - F_j(T)$ = Probability that the time to occurrence of the j^{th} major failure event will be greater than T.

and equivalently

$1 - F_j(T)$ = Probability that the number of major failure events in the time 0 to T is less than j.

Since the major failure events are happening in accordance with a Poisson probability law with an assumed mean rate λ, it follows that

$$1 - F_j(T) = \sum_{i=0}^{j-1} \frac{1}{i!} (\lambda T)^i e^{-\lambda T}$$

or

$$F_j(T) = \sum_{i=j}^{\infty} \frac{1}{i!} (\lambda T)^i e^{-\lambda T}$$

Therefore $$\int_0^T \frac{(\lambda t)^{j-1}}{\Gamma(j)} e^{-\lambda t}\, dt \equiv \sum_{i=j}^{\infty} \frac{1}{i!} (\lambda T)^i e^{-\lambda T}$$

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13 ABSTRACT

A complex system envisioned can accomplish a given mission with the aid of any one of several alternative structures. Each of these alternative structures (subsystems), although capable of mission accomplishment, exhibit different levels of performance due to its component inventory. The structures are ordered in preference accordingly. Operation on less-preferred structures is a function of component/structure failure histories in prior structures. Failures are classified by three levels of severity. A system effectiveness model is developed to provide a measure of effectiveness for the overall complex system which encompasses all possible alternative structures.

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